

Yau College Math Competition 2024
Final Probability and Statistics
Individual All-round Problems (June 8-9, 2024)

Choose **at least** 1 from the following 2 problems.

Problem 1.

(1) Suppose there is a biased coin with a probability p of landing heads. How can we use this coin to generate a Bernoulli(0.5) random variable?

(2) How many tosses are required on average to generate one random digit using this method?

(3) How can you use this biased coin to generate a uniform distribution on $\{1, 2, 3\}$ or, in general, a uniform distribution on $\{1, 2, \dots, n\}$?

Problem 2. We have n observations y_i , $i = 1, \dots, n$, satisfying the simple linear regression model:

$$y_i = \alpha + \beta x_i + \epsilon_i,$$

where ϵ_i are iid random variables with mean 0 and variance σ^2 . Let $\hat{\beta}$ denote the least squares estimator of β .

(1) Suppose $x_i = i$. Is $\hat{\beta}$ consistent? Asymptotically normal? Explain your answer.

(2) Answer the same question for $x_i = e^i$.