

## Problem for Individual Overall Contest

**Problem 1.** (1) Express the quotient ring

$$\mathbb{Z}[x_1, x_2, x_3]/(x_1x_2 + x_3^2 - 2)$$

as

$$\mathbb{Z}[x_1, \dots, x_n]/(f_1, \dots, f_m)$$

for some integers  $n$  and  $m$  such that

- (a)  $f_i$  are square-free polynomials;
- (b)  $\deg f \leq 2$ ;
- (c) all coefficients of  $f_i$  are equal to 1.

for all  $1 \leq i \leq k$ .

- (2) Can every quotient ring  $\mathbb{Z}[z_1, \dots, z_k]/I$  be expressed as  $\mathbb{Z}[x_1, \dots, x_n]/(f_1, \dots, f_m)$  satisfying the above three conditions as in (1)?