

**Yau College Math Competition 2022**  
**Final Probability and Statistics**  
**Individual Exam Problems (Aug. 20-21, 2022)**

**Problem 1.** Suppose that  $\xi, \eta$  are two random variables and there exists a function  $F$  such that

$$P(\xi \leq x, \eta \leq y) = F(x \wedge y).$$

Discuss the relation between  $\xi$  and  $\eta$ .

**Problem 2.** Let  $\xi$  be a standard Gaussian random variable. Prove that there exists  $C > 0$  such that

$$(\mathbb{E}|\xi|^p)^{\frac{1}{p}} \leq C\sqrt{p}, \quad \forall p > 1.$$

**Problem 3.** To order  $n$  random natural numbers  $\{x_1, x_2, \dots, x_n\}$  according to their magnitudes, pick randomly a number  $x$  from  $\{x_1, x_2, \dots, x_n\}$ , comparing all other numbers with  $x$ , place the smaller ones to the left of  $x$ , and the bigger ones to the right of  $x$ . Repeating the above procedure for the numbers to the left and right of  $x$ , respectively,  $\dots$ , until the numbers  $\{x_1, x_2, \dots, x_n\}$  are placed in an increasing order as  $x_{(1)} < x_{(2)} < \dots < x_{(n)}$ . Using probabilistic argument (e.g., conditional expectation) to evaluate roughly the average number of steps (each step is a comparing of two numbers) needed to complete the task for large  $n$ .