Syllabus on Geometry and Topology

Differential Geometry:

- Basics of smooth manifolds: Inverse function theorem, implicit function theorem, submanifolds, Sard’s Theorem, embedding theorem, transversality, degree theory, integration on manifolds.
- Basics of matrix Lie groups over R and C: The definitions of \( \text{Gl}(n), \text{SU}(n), \text{SO}(n), \text{U}(n) \), their manifold structures, Lie algebras, right and left invariant vector fields and differential forms, the exponential map.
- Definition of real and complex vector bundles, tangent and cotangent bundles, basic operations on bundles such as dual bundle, tensor products, exterior products, direct sums, pull-back bundles.
- Definition of differential forms, exterior product, exterior derivative, de Rham cohomology, behavior under pull-back.
- Metrics on vector bundles.
- Riemannian metrics, definition of a geodesic, existence and uniqueness of geodesics.
- Definition of a principal Lie group bundle for matrix groups.
- Associated vector bundles: Relation between principal bundles and vector bundles.
• Definition of covariant derivative for a vector bundle and connection on a principal bundle. Relations between the two.

• Definition of curvature, flat connections, parallel transport.

• Definition of Levi-Civita connection and properties of the Riemann curvature tensor, manifolds of constant curvature.

• Jacobi fields, second variation of geodesics

• Manifolds of nonpositive curvature, manifolds of positive curvature


Algebraic Topology:

• Fundamental groups

• Covering spaces

• Higher homotopy groups

• Fibrations and the long exact sequence of a fibration

• Singular homology and cohomology

• Relative homology
• CW complexes and the homology of CW complexes
• Mayer-Vietoris sequence
• Universal coefficient theorem
• Kunneth formula
• Poincare duality
• Lefschetz fixed point formula
• Hopf index theorem
• Cech cohomology and de Rham cohomology.
• Equivalence between singular, Cech and de Rham cohomology