Syllabus on Geometry and Topology

Differential Geometry:

- Basics of smooth manifolds: Inverse function theorem, implicit function theorem, submanifolds, Sard's Theorem, embedding theorem, transversality, degree theory, integration on manifolds.
- Basics of matrix Lie groups over R and C: The definitions of Gl(n), SU(n), SO(n), U(n), their manifold structures, Lie algebras, right and left invariant vector fields and differential forms, the exponential map.
- Definition of real and complex vector bundles, tangent and cotangent bundles, basic operations on bundles such as dual bundle, tensor products, exterior products, direct sums, pull-back bundles.
- Definition of differential forms, exterior product, exterior derivative, de Rham cohomology, behavior under pull-back.
- Metrics on vector bundles.
- Riemannian metrics, definition of a geodesic, existence and uniqueness of geodesics.
- Definition of a principal Lie group bundle for matrix groups.
- Associated vector bundles: Relation between principal bundles and vector bundles

- Definition of covariant derivative for a vector bundle and connection on a principal bundle. Relations between the two.
- Definition of curvature, flat connections, parallel transport.
- Definition of Levi-Civita connection and properties of the Riemann curvature tensor, manifolds of constant curvature.
- Jacobi fields, second variation of geodesics
- Manifolds of nonpositive curvature, manifolds of positive curvature

References: V. Guillemin, A. Pollack, Differential topology; J. Milnor, Topology from the differentiable viewpoint; Cliff Taubes: Differential geometry: Bundles, Connections, Metrics and Curvature; John Lee: Introduction to Riemannian manifolds, second edition; S. Kobayashi and K. Nomizu: Foundations of Differential Geometry.

Algebraic Topology:

- Fundamental groups
- Covering spaces
- Higher homotopy groups
- Fibrations and the long exact sequence of a fibration
- Singular homology and cohomology
- Relative homology

- CW complexes and the homology of CW complexes
- Mayer-Vietoris sequence
- Universal coefficient theorem
- Kunneth formula
- Poincare duality
- Lefschetz fixed point formula
- Hopf index theorem
- Cech cohomology and de Rham cohomology.
- Equivalence between singular, Cech and de Rham cohomology

References: Alan Hatcher: Algebraic Topology; William Fulton: Algebraic Topology; Edwin Spanier: Algebraic Topology; M. Greenberg and J. Harper: Algebraic Topology: A First Course.